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PLANT MODERNIZATION UNDER CONDITIONS OF COMPETITION AND TECHNOLOGICAL IMPROVEMENT

R. Kalaba, A. Kent and M. Prestrud

PREPARED FOR:
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The RAND Corporation
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PREFACE

As part of its continuing research effort for the Air Force, The RAND Corporation has been carrying out extensive investigations in the fields of equipment replacement and plant modernization. Such studies contribute toward an efficient industrial base for the Air Force. They also provide concepts applicable to the Air Force's continuing process of force modernization.

This Memorandum deals with two important aspects of plant modernization: (1) technological improvement and (2) competition. The authors show how to formulate such problems in mathematical terms, and bring the power of modern computing machines to bear on these aspects of the problem. This Memorandum should be of particular interest to Air Force contracting officers and to planning personnel.

SUMMARY

Two important aspects of plant modernization are (1) technological improvement and (2) plans of competitors for modernization of their plants. The purpose of this paper is to present a novel mathematical model of plant modernization in which both of these factors are considered.

In addition to the basic equations themselves, a FORTRAN program for their resolution is given, and the results of some numerical experiments are presented.

The primary aim of a mathematico-economical study such as this is to provide a flexible mathematical tool for determining the sensitivity of optimal decision policies to changes in basic assumptions for the physical situation. It is hoped that this will lead to increased understanding of actual plant modernization problems.

ACKNOWLEDGMENTS

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CONTENTS

I. INTRODUCTION

Plant modernization--based on a desire to take full advantage of the fruits of current technology--represents both an opportunity and a hazard to the manufacturing firm. On the one hand there is the promise of increased economies in production, and on the other there is the large capital expenditure involved. In a situation complicated and fraught with uncertainties, the single most important factor may be a competitor's decision as to whether or not he will modernize -- a decision which is beyond one's own control.

Developments of the last decade--including the advent of the high-speed digital computer and the cultivation of a new mathematical field, dynamic programming[1]--provide management with tools which may aid in the formulation and resolution of decision problems. Our aim is to indicate the formulation and computational treatment of some problems in plant modernization, with emphasis on competition and technological improvement. In this latter respect the treatment is novel and original. Earlier discussions of plant modernization, where additional references may be found, are available in Refs. 1-5, 9.

II. FORMULATION

Let us consider that the present plant was purchased in year T and that it is currently t years old (the current year is, of course, T + t). We may consider both the year of purchase and the present age as averages if the plant was purchased over a period of time. We characterize this equipment and the effects of competition by defining the two functions $n_1(T,t)$ and $n_2(T,t)$:

(1) $n_1(T,t)$ = the profit (net return) from the next year of operation of equipment that was purchased in year T and is currently t years old, competition being heavy,

and

(2) $n_2(T,t)$ = the profit from the next year of operation of equipment that was purchased in year T and is currently t years old, competition being normal.*

The intensity of competition, here described by the two words "normal" and "heavy," is measured in terms of its effect on the return function. "Heavy competition" implies that the competitor has modernized his plant, and/or increased his promotional effort, etc., so as to reduce the profit-effectiveness of the subject corporation. "Normal competition" implies that the competitor has not yet modernized his plant.

Aside from the effect of competition, the return functions will be monotone decreasing functions of the age of the equipment, t. To reflect technological improvement, they will be monotone increasing in the year of purchase, T.

*The functions $n_i(T,t)$ may be defined, in a given application, in other but related ways, e.g., in terms of net receipts, change in net worth, etc.

Furthermore, reflecting the decreased profits resulting from heavy competition, we shall have

$$(3) \qquad n_1(T,t) < n_2(T,t)$$

In the event that competition has been normal, we shall assume that there is a probability p that competition will become heavy during the next year, i.e., that competitors will modernize (or make other major improvements in their effectiveness).

The purchase price of new equipment is denoted by c, and the salvage value of the old equipment by r. For simplicity we assume that these are constants.

In order to refer future gains to a current worth, we introduce a discount factor, a, which reflects the applicable rate of interest. Generally speaking, we shall have

$$(4) \qquad 0 < a < 1$$

Finally, let us consider that we wish to plan to operate over a period extending N years into the future; that is, the planning horizon of the subject corporation is N years ahead. Furthermore, we assume that when a competitor modernizes his plant so that competition becomes "heavy" it remains "heavy" during the rest of the process.

III. THE BASIC EQUATIONS

We can formulate our basic equations by introducing the two optimal return functions:

(1) $f_N(T,t)$ = the expected return from a process of duration N years, beginning with equipment that was purchased in year T, is t years old, competition having been heavy, and using an optimal plant modernization policy.

(2) $g_N(T,t)$ = the expected return from a process of duration N years, beginning with equipment that was purchased in year T, is t years old, competition having been normal, and using an optimal plant modernization policy.

Then, employing Bellman's principle of optimality[1], we find the relations

$$f_N(T,t) = \text{Max} \begin{bmatrix} n_1(T,t) + a\, f_{N-1}(T,t+1) \\ r - c + n_1(T+t,0) + a\, f_{N-1}(T+t,1) \end{bmatrix} \tag{3}$$

(4) $g_N(T,t) =$

$$\text{Max} \begin{bmatrix} p\left[n_1(T,t) + a\, f_{N-1}(T,t+1)\right] + (1-p)\left[n_2(T,t) + a\, g_{N-1}(T,t+1)\right] \\ r-c+p\left[n_1(T+t,0) + a\, f_{N-1}(T+t,1)\right] + (1-p)\left[n_2(T+t,0) + a\, g_{N-1}(T+t,1)\right] \end{bmatrix}$$

In general we cannot expect to resolve these equations analytically, which would tell us the correct decision to make under any set of circumstances and also tell us what the maximum expected profit during the remainder of the process is. Consequently, we

must turn to a computational treatment. The next section is devoted to some results aimed at showing how the optimal decision, keep or replace, depends upon various parameters of the process.

IV. COMPUTATIONAL RESULTS

To investigate the implications of the assumptions of the earlier sections we prepared a general FORTRAN program for use on an IBM 7090 computing machine. This program and a sample printout are given in the Appendix. For our computations we made the assumption that the annual profit functions have the forms

$$n_1(T,t) = h\, n_2(T,t) \tag{1}$$

$$n_2(T,t) = \left(A + B\,(T-T_o)\right) \exp\left[-\,t/\left(C + D(T-T_o)\right)\right] \tag{2}$$

where A, B, C, and D are four parameters. The constant T_o is the minimal model year we wish to consider. Under heavy competition the annual return for a year of operation is only h times the annual return for a year of operation under normal conditions ($h < 1$). The parameters A and C are measures of the productivity of equipment purchased in year T_o and of the durability of the equipment. The parameters B and D are measures of the improvements in the equipment to be expected through the purchase of later models of the equipment.

In our first calculation we used the values listed below.

(3)

$A = 500{,}000$	$T_o = 1945$	$p = 0, .5, 1.0$
$B = 100{,}000$	$c = 4{,}000{,}000$	
$C = 5$	$r = 2{,}000{,}000$	
$D = 1$	$a = .9$	
	$h = .7$	

We found that if there are ten or more years remaining, for the particular values of the parameters A, B, C, etc., selected, then the optimal decision concerning keeping or replacing the equipment is the same as if infinitely many stages remained. Thus, if the planning horizon is ten or more years in the future, we are effectively in a steady-state decision region. In fact, we see from the following graphs that the optimal decision, for the case being considered, consists essentially in replacing equipment that is five years old or older and keeping equipment that is younger than five years old. This is true regardless of whether our estimate of the parameter p is 0.0, 0.5, or 1.0. It will be recalled that p is our estimate of the probability that the competitor will modernize his plant during the coming year, assuming that he has not yet done so. The exact results are given in the graphs (Figs. 1-5).

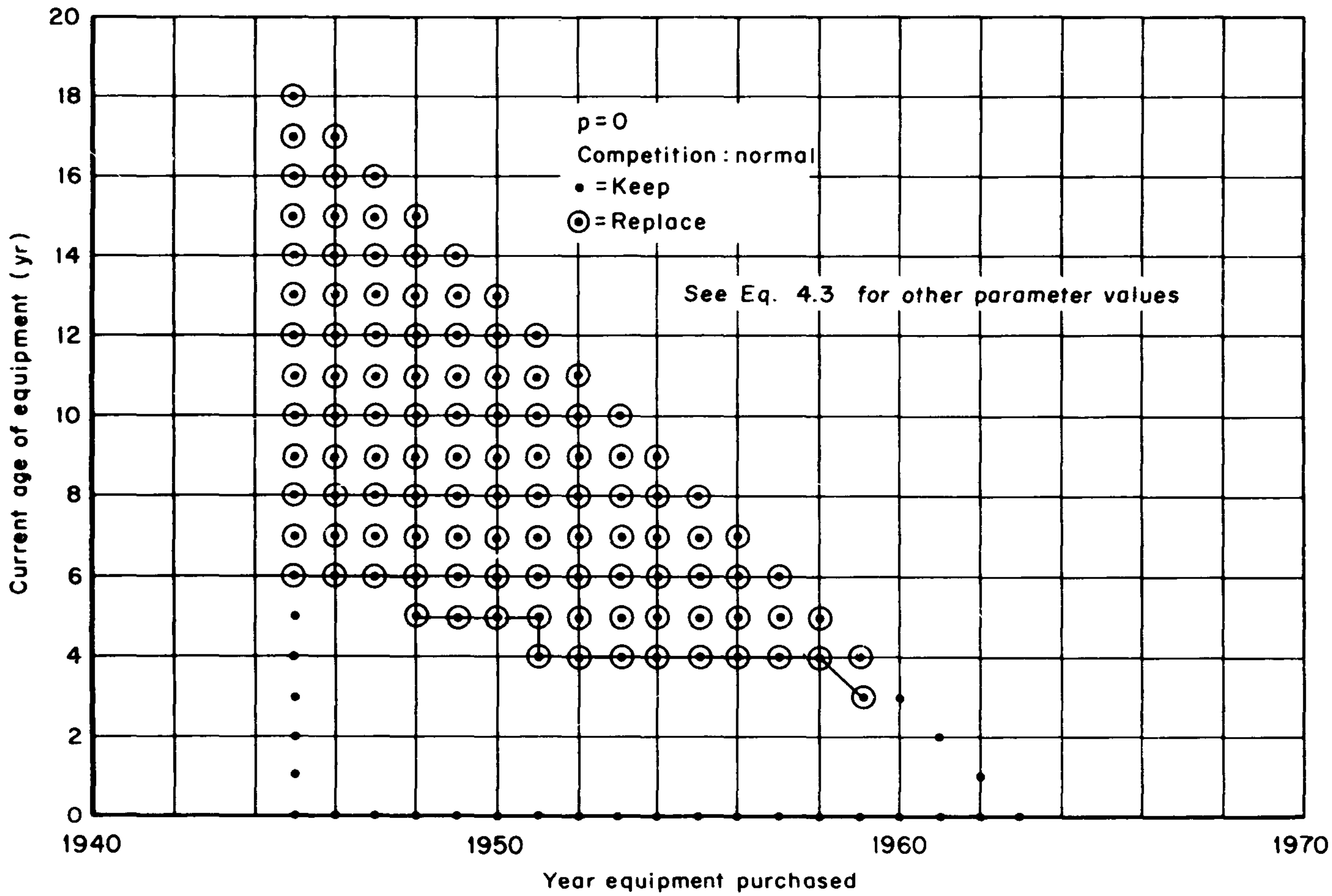

Fig. 1 — Some optimal steady-state decisions

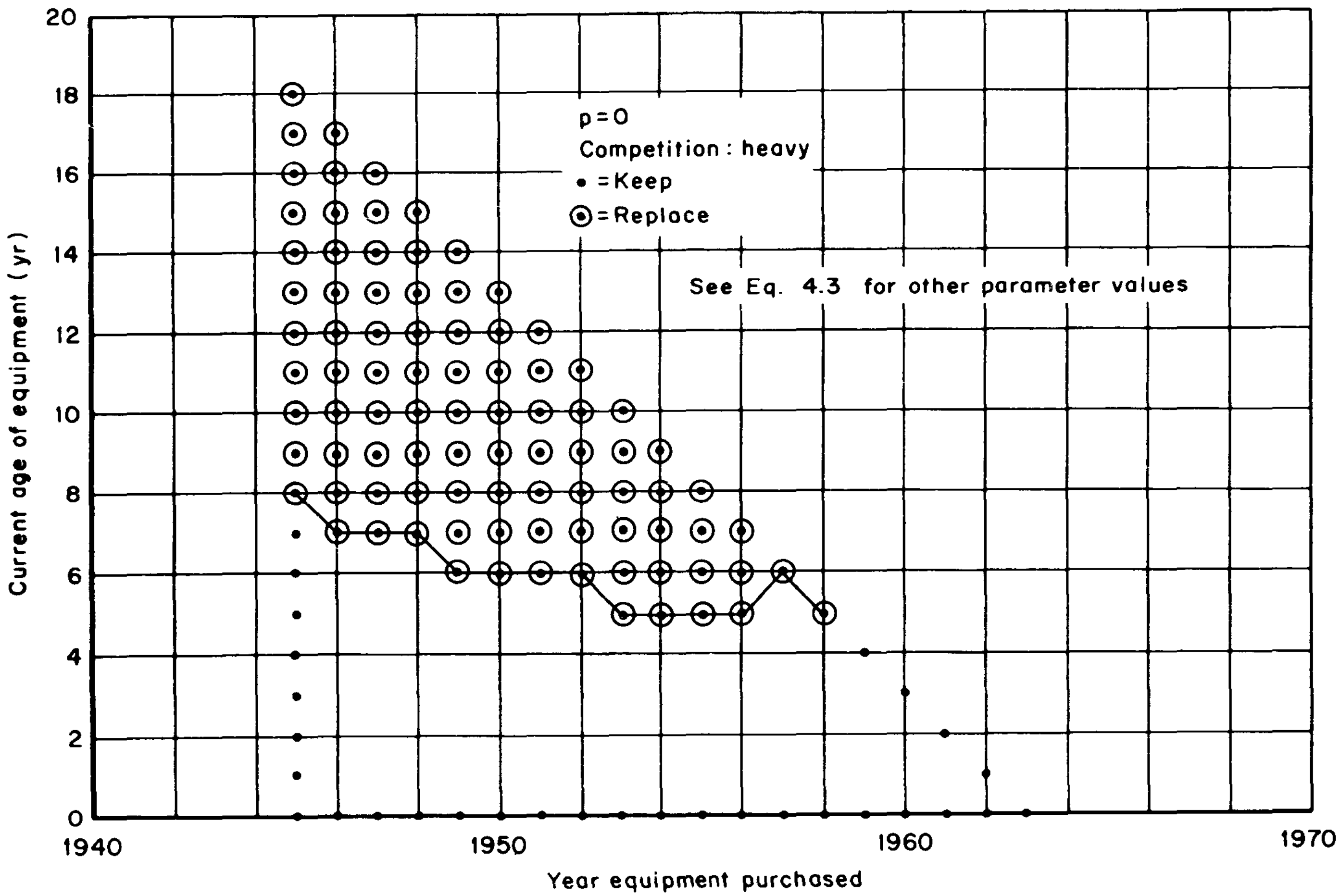

Fig. 2 — Some optimal steady-state decisions

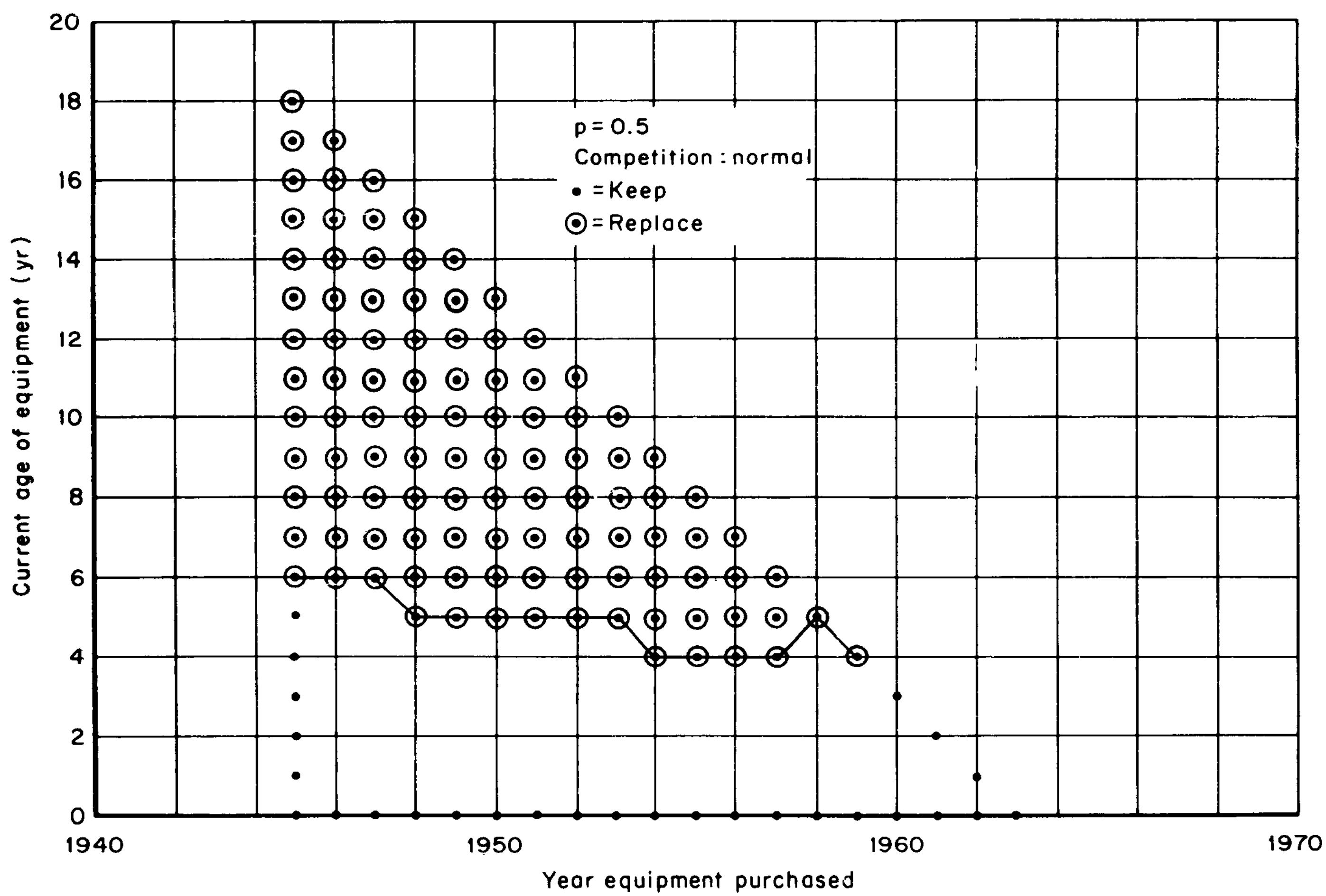

Fig. 3 — Some optimal steady-state decisions

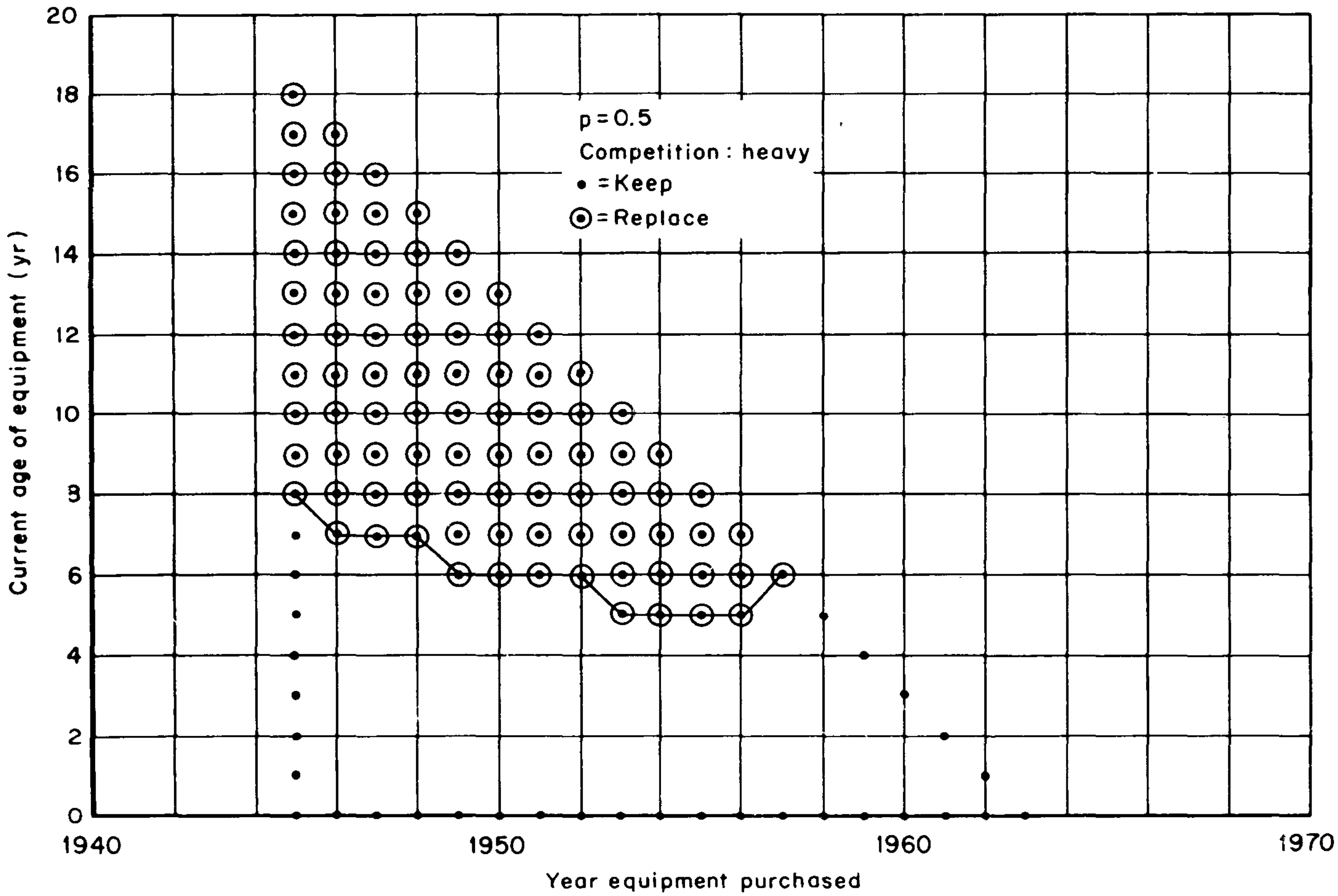

Fig. 4 — Some optimal steady-state decisions

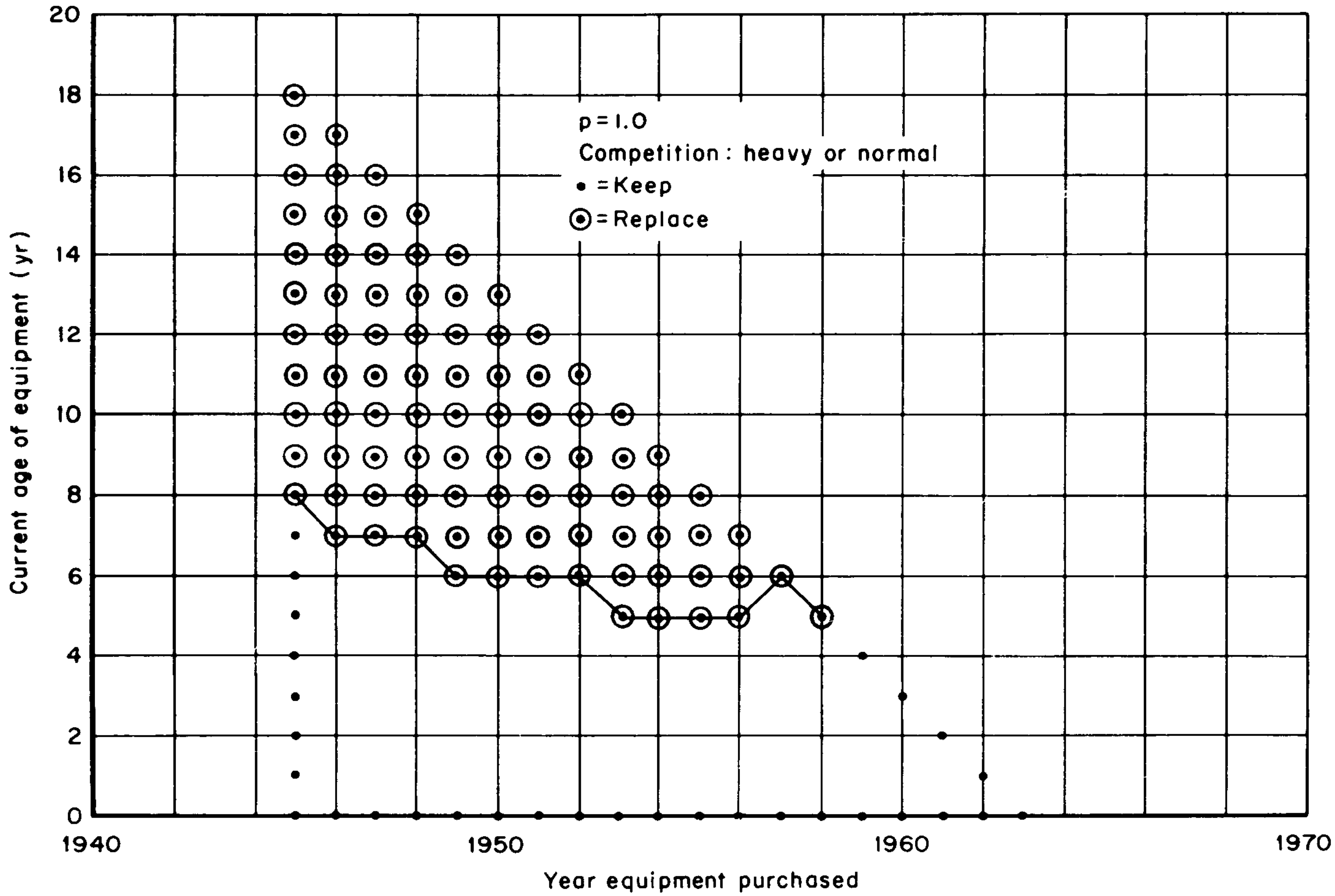

Fig. 5 — Some optimal steady-state decisions

Now let us consider a new physical situation in which the resale value has been reduced from two million to 500,000, the parameter B from 100,000 to 50,000 and the parameter D from 1 to 0.5. Under these circumstances the purchase of new equipment looks much less attractive than above. The results as to the correct decision to make, providing the planning horizon is ten or more years in the future, are shown in the graphs which follow. In particular we see that for these given parameter values, there is some dependence on both the initial level of competition and the value of p, the probability that the competition will convert to heavy competition if it has not yet done so. Notice, as is shown in the following graphs, that now it may be optimal to keep equipment that is ten years old or even older (Figs. 6-10).

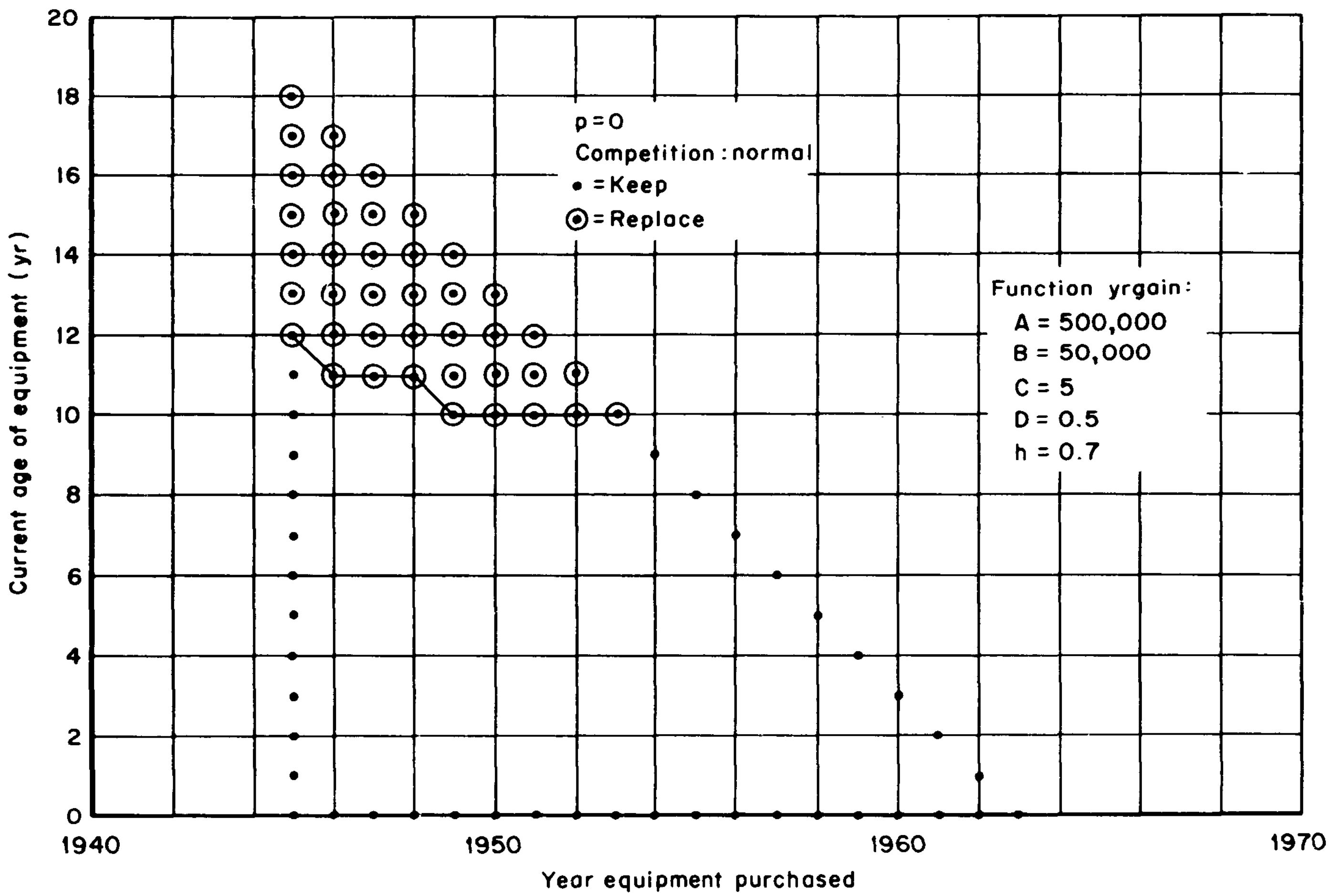

Fig. 6 — Some optimal steady-state decisions

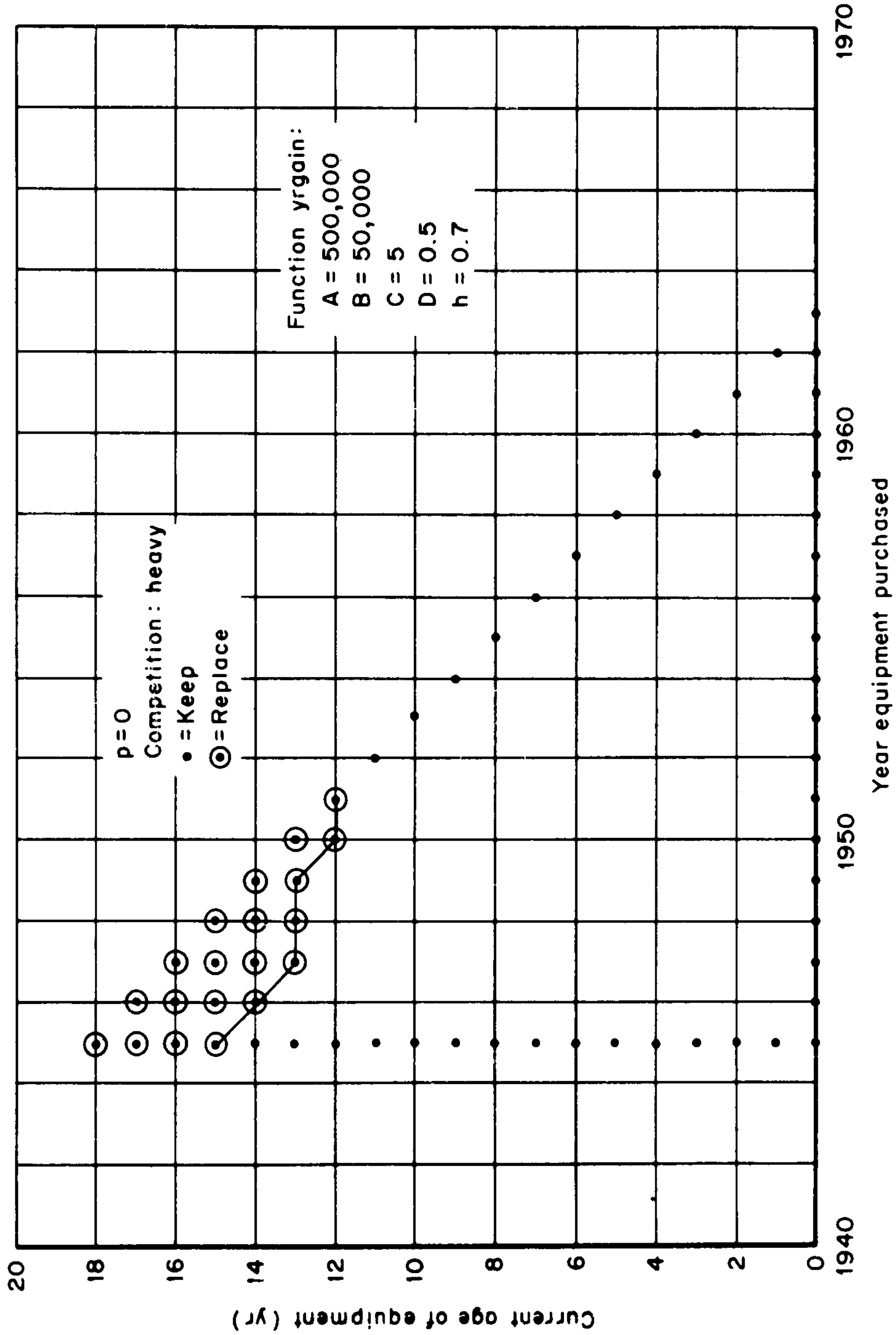

Fig. 7 — Some optimal steady-state decisions

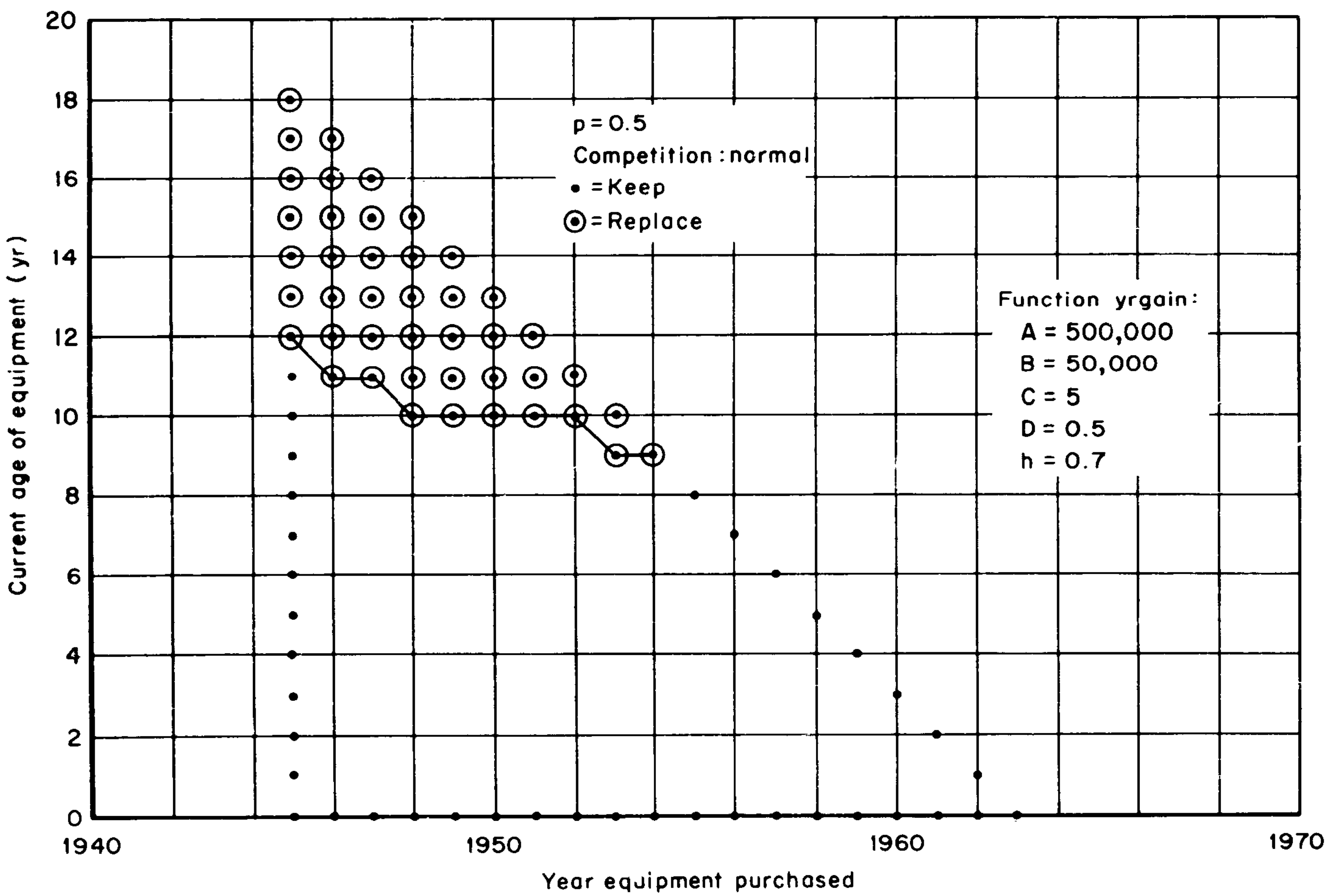

Fig. 8 — Some optimal steady-state decisions

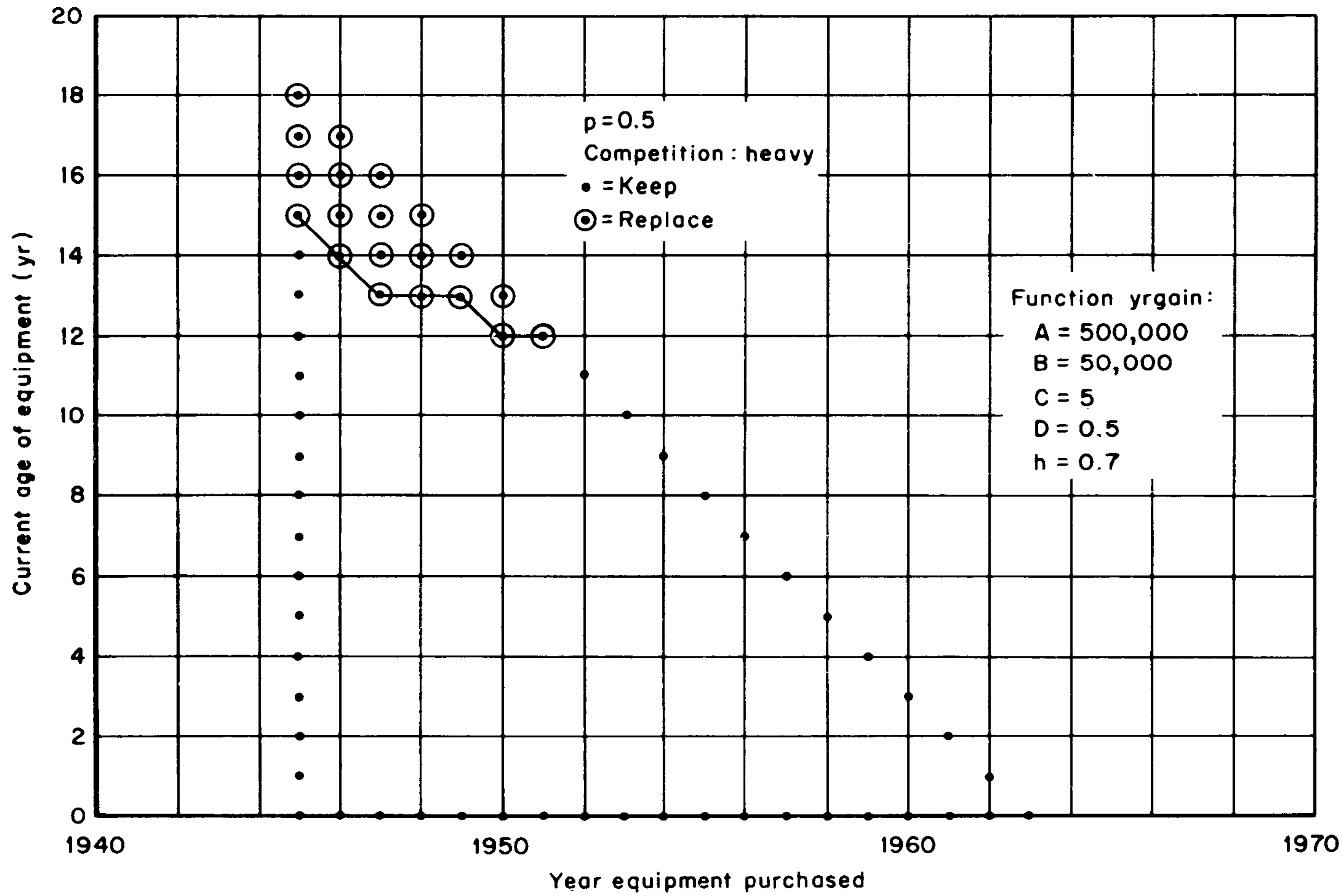

Fig. 9 — Some optimal steady-state decisions

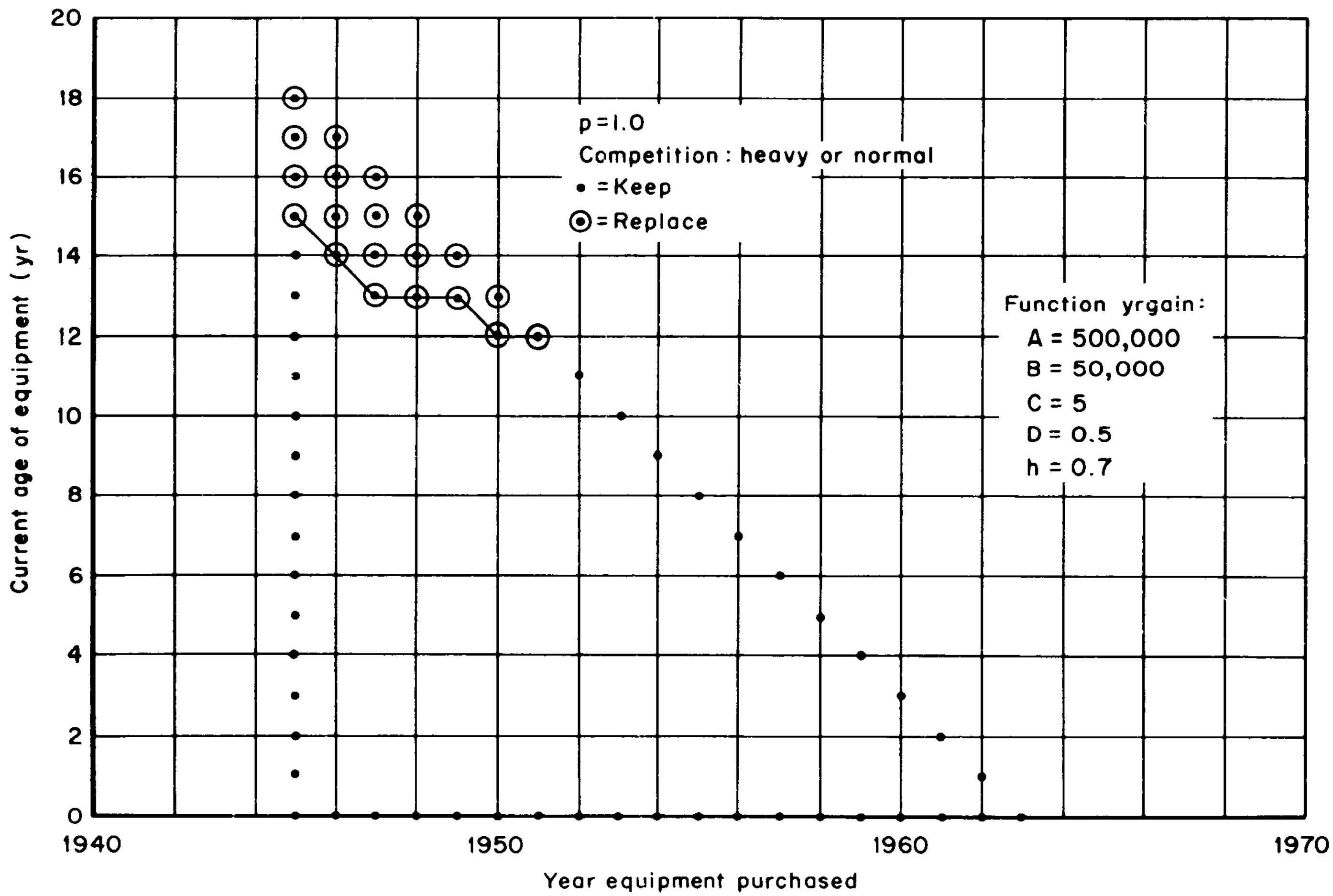

Fig. 10 — Some optimal steady-state decisions

Finally, to reflect the more volatile conditions found in certain industries regarding innovation and share-of-market, the value of the parameter h was reduced arbitrarily to 0.2. Thus, the annual return is subject to a very large reduction by a competitor's decision to modernize. In examining the next set of figures we see that as p (the probability that the competition will convert to heavy competition) increases, there is a tendency toward keeping the current equipment (Figs. 11-15).

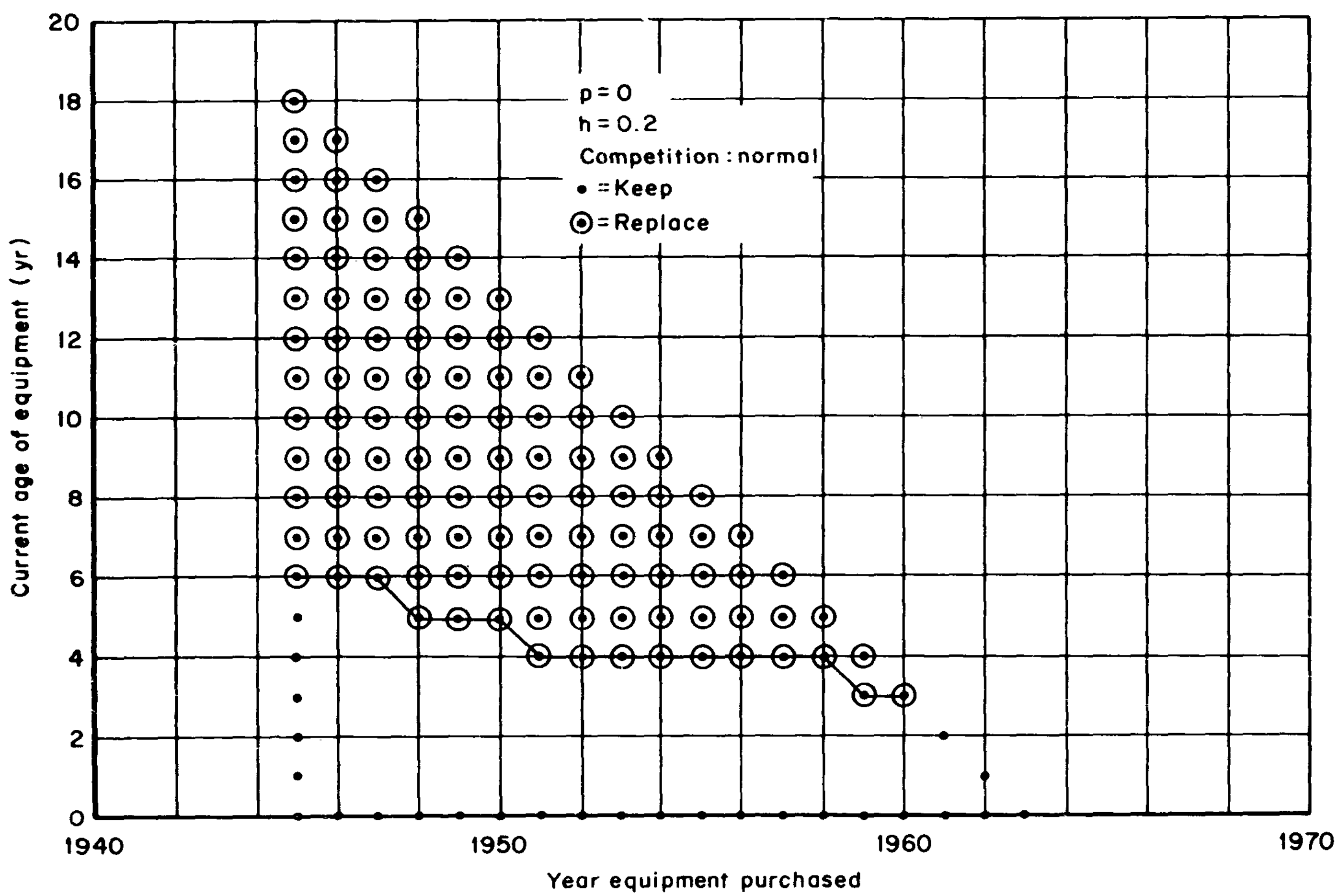

Fig. 11 — Some optimal steady-state decisions

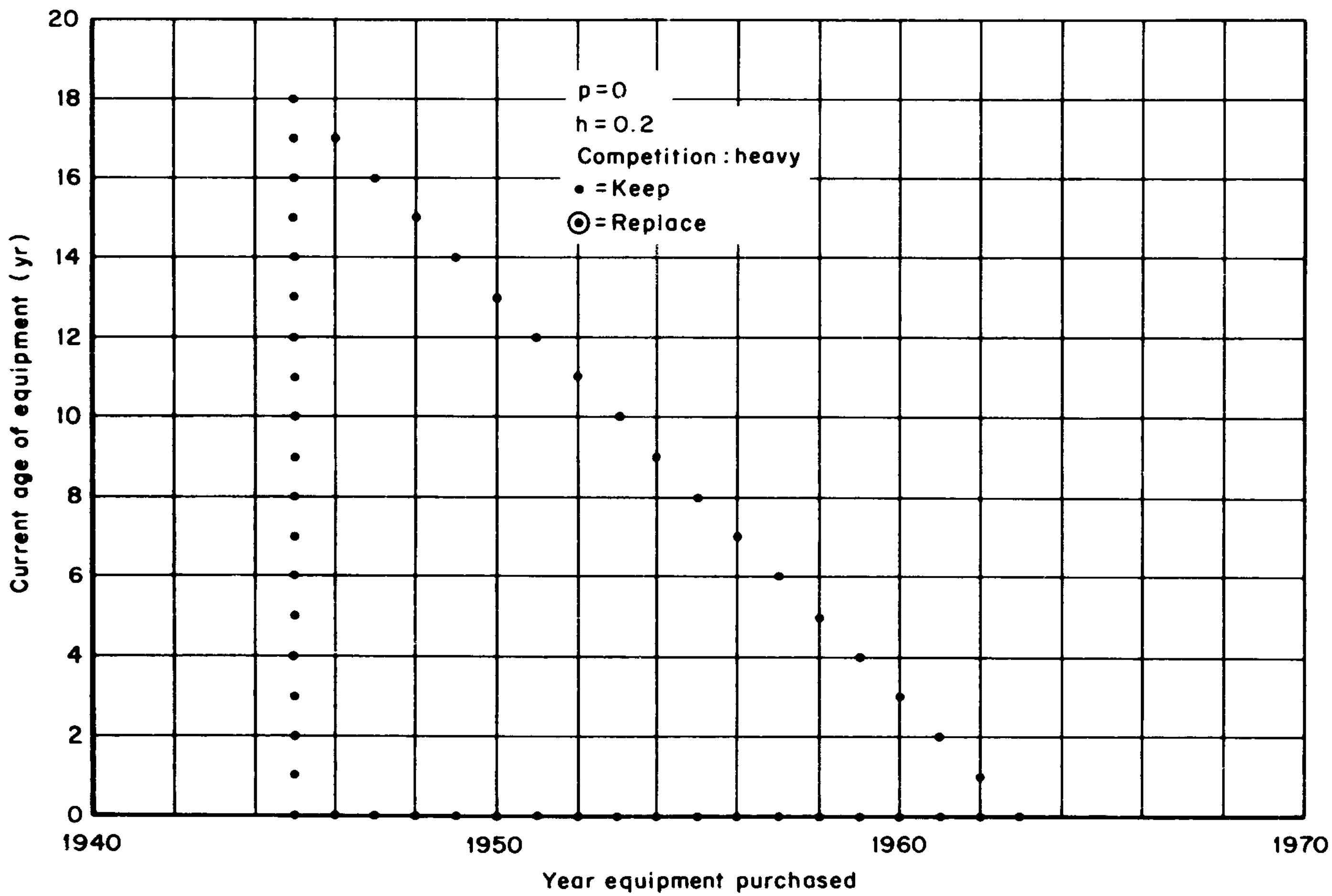

Fig. 12 — Some optimal steady-state decisions

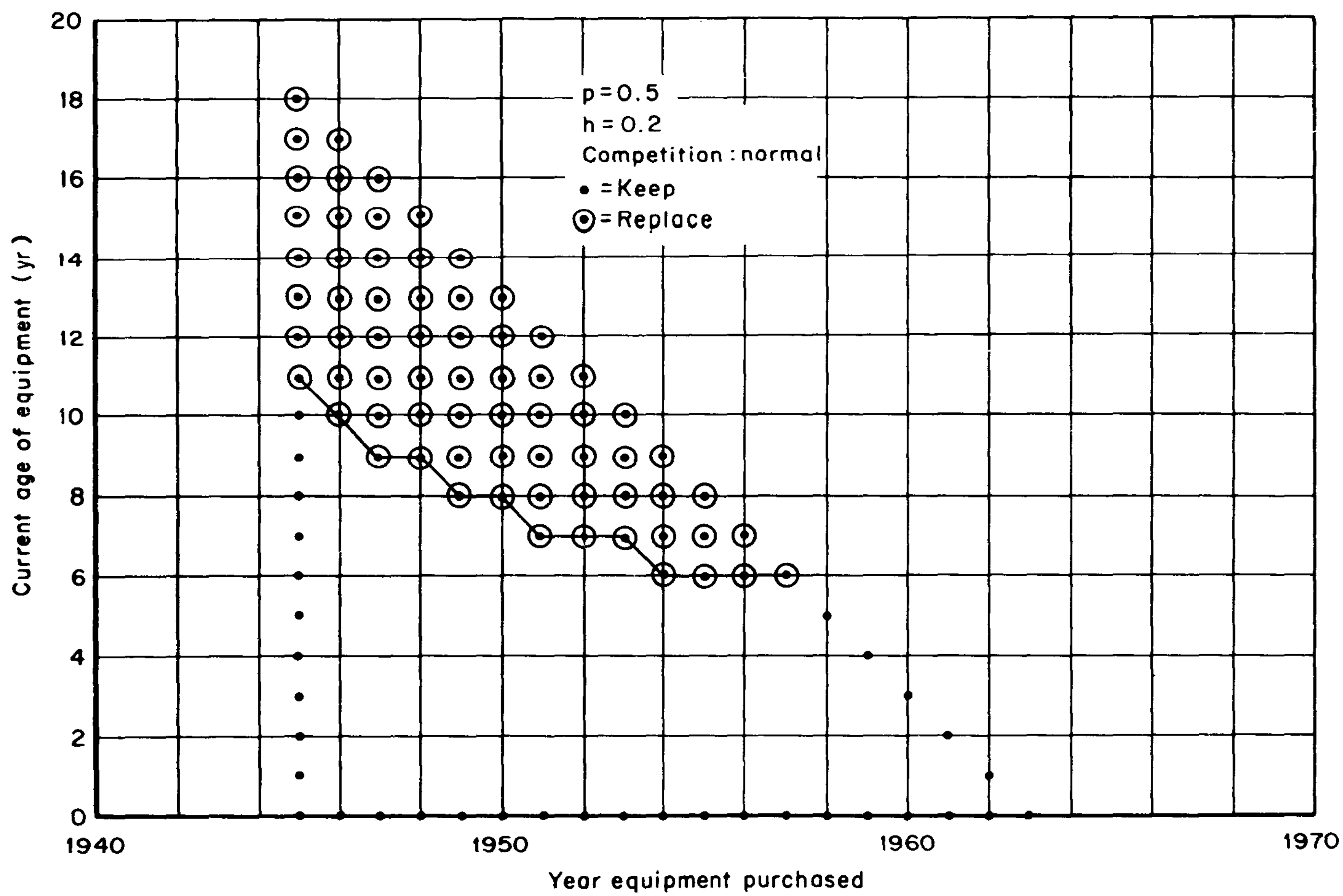

Fig. 13 — Some optimal steady-state decisions

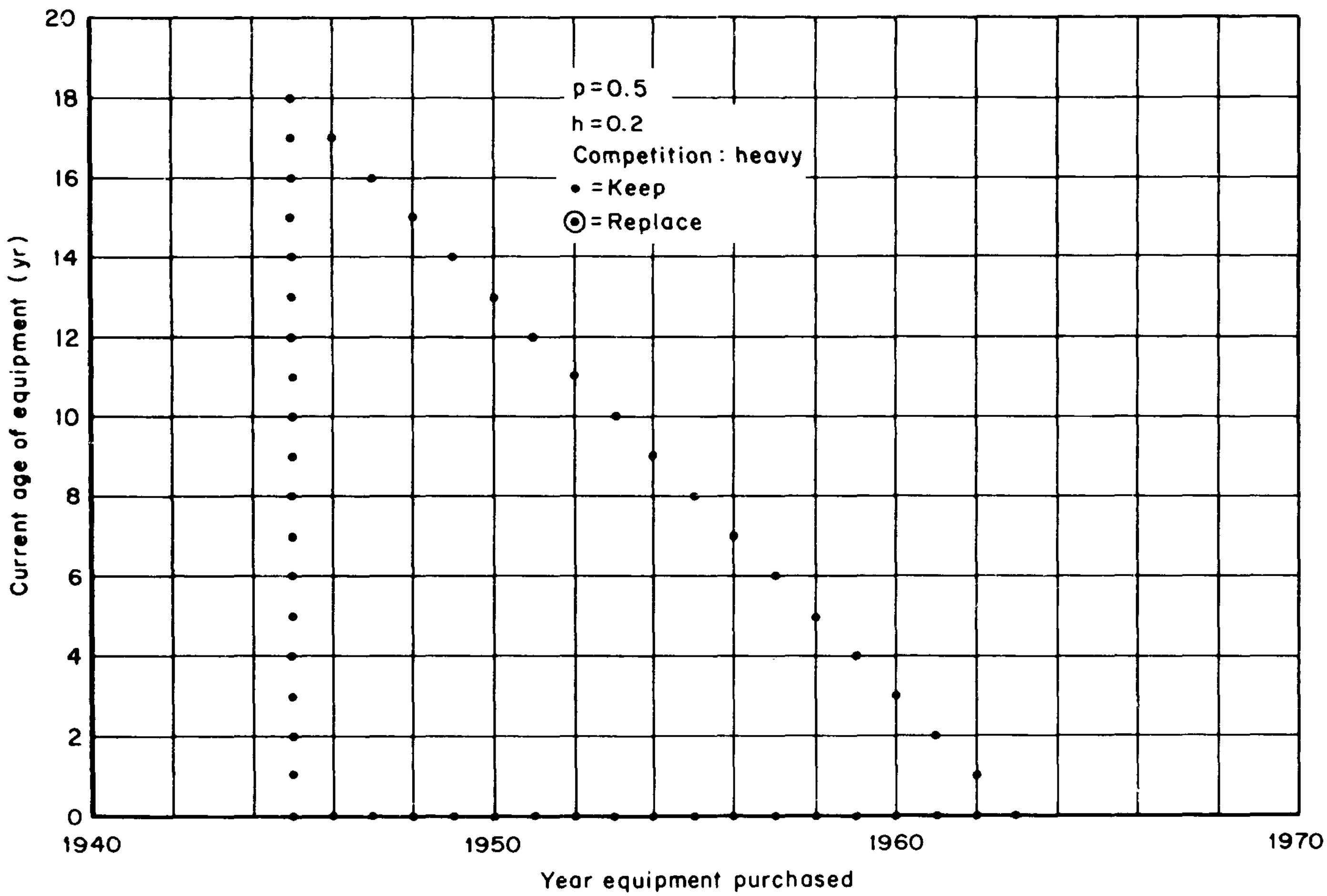

Fig. 14 — Some optimal steady-state decisions

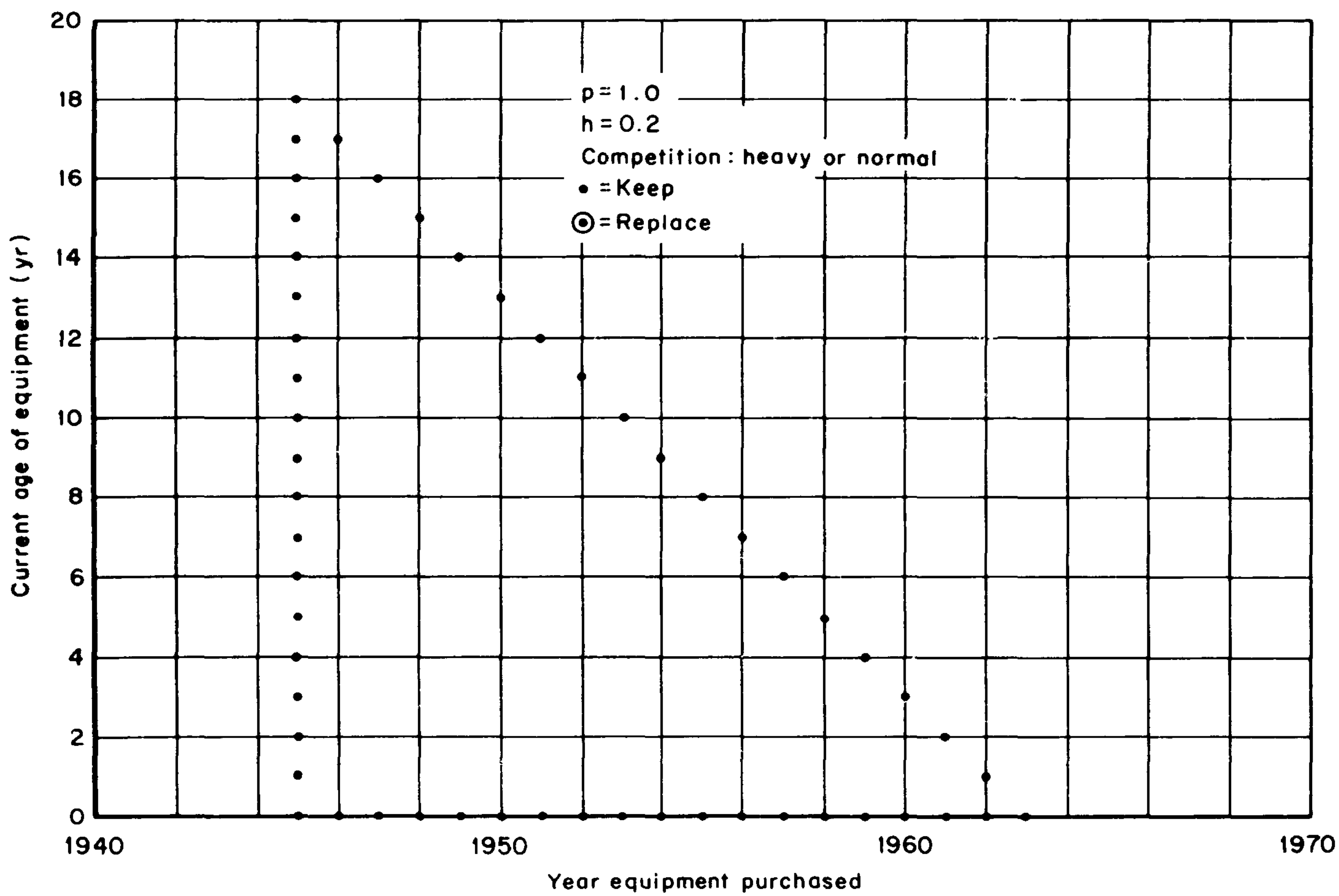

Fig. 15 — Some optimal steady-state decisions

Let us once again remark that the foregoing examples are merely illustrative. They show the ease with which the dependence of the optimal decision on various parameters may be ascertained.

V. DISCUSSION

The return functions, $n_i(T,t)$ are clearly pivotal and, in actual application, uncertain. The inherent computational efficiency of the dynamic programming technique permits the practical investigation of sensitivities to various parameters. This is important, since many parameters of the problem represent only informed estimates and expert opinion available to the decision-maker.

Based on their investigations, the authors suggest the following areas as probably fruitful for further work:

<u>Objective Functions</u>:

Under what circumstances should the objective be other than maximizing expected profits over the planning period? How could such aspects as risk-taking and financing considerations be included?

<u>The Parameter p</u>:

In our model, p was assumed to be a constant, independent of the year. Elaboration was considered generally unwarranted in view of the uncertainties in the estimation of p. Similarly, the effect of the subject company's modernization *per se* on p, i.e., influencing competitor action, was neglected on the grounds that a major change in competition would take at least a year to be effective, and that the planning process would be repeated annually. However, it would be instructive to look further at these matters.

Competition Levels:

Future work might well include considerations of more than two discrete levels of competition, as well as the case in which the intensity or effectiveness of competition may vary continuously over an interval.

Planning Horizon:

The assumption that competition once "heavy" remains so, becomes increasingly unrealistic as N increases. But the results of this investigation (see figures) were for the "steady-state" ($N \geq 10$). Is this a suitable planning horizon?

Computational Considerations:

Along mathematical lines, let us point out that if a more realistic description of the state of the system is given, in which more than the year of purchase and the current age are given, we are confronted with dimensionality difficulty even with the use of the dynamic programming technique. The use of polynomial approximation[7] may be useful in this circumstance. We are frequently interested in the steady-state optimal decisions, i.e., the decisions to be made when the planning horizon is far in the future. In the usual approach this necessitates considering processes of duration one, two, three, etc., until a duration is reached for which no further changes in the decisions, as functions of the duration of the process, occur. To speed up the computing process we may use the results for processes of short duration, in conjunction with various prediction schemes, to estimate the optimal steady-state behavior. This is discussed in Ref. 8.

Appendix

THE FORTRAN PROGRAM AND A SAMPLE PRINTOUT

Pages 29 through 36 are FORTRAN listings of the main program and the one subroutine used in solving this problem. The variables are defined in the comments.

Statements 50, 53, 55 and 57 begin respectively the computing DØ-loops for process duration, model year, age of equipment, and type of competition. The DØ-loop of statements 74-77 is a shifting operation enabling retention in the computer's memory of only items pertaining to a single stage of the process at a time.

It should be noted in the output for process of duration one year and equipment purchased in 1945, say, that as the present age of equipment ranges from 0 to 27 years, the calendar year under consideration ranges from 1945 to 1972.

```
C     CORPORATION EQUIPMENT REPLACEMENT MULTISTAGE DECISION PROBLEM
C
C     IN = INPUT TAPE REEL NUMBER
C     IOUT = OUTPUT TAPE REEL NUMBER
C     NMAX = NUMBER OF STAGES IN PROCESS POSSIBLE
C     N = CURRENT STAGE
C     NCTOFF = NUMBER OF STAGES IN PROCESS ACTUALLY COMPUTED
C     MY = AVERAGE YEAR IN WHICH PRESENT EQUIPMENT PURCHASED
C     MYMIN = EARLIEST AVERAGE PURCHASE YEAR CONSIDERED FOR PROCESS
C     MYMAX = LATEST AVERAGE PURCHASE YEAR CONSIDERED,VARIES WITH STAGE
C     NMY = YEAR IN WHICH NEW EQUIPMENT PURCHASED
C     IT = AGE OF EQUIPMENT AT CURRENT STAGE
C     ITT = AGE OF EQUIPMENT AT STAGE N-1
C     ITTMAX = HIGHEST AGE OF STAGE N-1 EQUIPMENT
C     K=1,LEVEL OF COMPETITION HAS BEEN HEAVY
C     K=2,LEVEL OF COMPETITION HAS BEEN NORMAL
C     PI = PROBABILITY COMPETITOR WILL RESORT TO HEAVY COMPETITION AT A
C          GIVEN STAGE ($ THENCEFORTH)
C     R = RESALE VALUE OF OLD EQUIPMENT
C     P = PURCHASE PRICE OF NEW EQUIPMENT
C     A = CURRENT VALUE OF A DOLLAR ONE YEAR HENCE
C     F(MY,IT) = MAXIMUM GAIN FROM STAGE N TO END OF PROCESS WHEN
C                COMPETITION HAS BEEN HEAVY
C     G(MY,IT) = MAXIMUM EXPECTED GAIN FROM STAGE N TO END OF PROCESS
C                WHEN COMPETITION HAS BEEN NORMAL
C     FF(MY,IT) = MAXIMUM GAIN FROM STAGE N-1 TO END OF PROCESS WHEN
C                 COMPETITION HAS BEEN HEAVY
C     GG(MY,IT) = MAXIMUM EXPECTED GAIN FROM STAGE N-1 TO END OF PROCESS
C                 WHEN COMPETITION HAS BEEN NORMAL
C     FKEEP = GAIN FROM STAGE N TO END OF PROCESS IF KEEP EQUIPMENT WHEN
C             COMPETITION HAS BEEN HEAVY
C     FREPL = GAIN FROM STAGE N TO END OF PROCESS IF REPLACE EQUIPMENT
C             WHEN COMPETITION HAS BEEN HEAVY
C     GKEEP = EXPECTED GAIN FROM STAGE N TO END OF PROCESS IF KEEP
C             EQUIPMENT WHEN COMPETITION HAS BEEN NORMAL
C     GREPL = EXPECTED GAIN FROM STAGE N TO END OF PROCESS IF REPLACE
C             EQUIPMENT WHEN COMPETITION HAS BEEN NORMAL
C     PROFIT = F(MY,IT) OR G(MY,IT)
C     FUNCTION YRGAIN(MY,IT,K) = GAIN FOR CURRENT YEAR ALONE
C     ACAP,B,C,D = CONSTANTS FOR FUNCTION YRGAIN
C     FR = FRACTION OF NORMAL PROFITS IN FACE OF INCREASED COMPETITION
C
10    FORMAT (6I12)
11    FORMAT (6E12.8)
121   FORMAT (8HONMAX = ,I2)
1211  FORMAT (10H NCTOFF = ,I2)
122   FORMAT (9H MYMIN = ,I4)
123   FORMAT (6H PI = ,E15.8,5H,R = ,E15.8,5H,P = ,E15.8,5H,A = ,E15.8)
124   FORMAT (8H ACAP = ,E15.8,5H,B = ,F15.8,5H,C = ,F15.8,5H,D = ,
     X        E15.8,6H,FR = ,E15.8)
151   FORMAT (1H1,22X,26HFOR PROCESSES OF DURATION ,I2,6H YEARS)
152   FORMAT (/1HO,4X,33H------------ STATES -------------,14X,7HOPTIMAL
     1,6X,7HOPTIMAL/
     2        1H ,51X,8HDECISION,5X,8HEXPECTED/
     3        17H AVE.YEAR PRESENT,3X,11HPRESENT AGE,3X,8HLEVEL OF,23X,
```

CCRPRE

```
     46HRETURN/
     5     '    18H EQUIP'T.PURCHASED,2X,11HOF EQUIP'T.,2X,11HCOMPETITION/
     6/)
153   FORMAT (1H ,4X,I4)
154   FORMAT (1H+,21X,I2)
155   FORMAT (1H+,34X,5HHEAVY)
156   FORMAT (1H+,34X,6HNORMAL)
157   FORMAT (1H+,51X,4HKEEP)
158   FORMAT (1H+,51X,7HREPLACE)
159   FORMAT (1H+,64X,1PE9.2/1H )
C
      DIMENSION F(50,50),G(50,50),FF(50,50),GG(50,50)
      COMMON ACAP,B,C,D,FR,MYMIN
C
20    IN = 41
21    ICUT = 42
22    INPUT IN,10,NMAX,NCTOFF,MYMIN
23    INPUT IN,11,PI,R,P,A
24    INPUT IN,11,ACAP,B,C,D,FR
30    OUTPUT ICUT,121,NMAX
31    OUTPUT ICUT,1211,NCTOFF
32    OUTPUT ICUT,122,MYMIN
33    OUTPUT ICUT,123,PI,R,P,A
34    OUTPUT ICUT,124,ACAP,B,C,D,FR
C
5001  DO 5004 I=1,50
5002  DO 5004 M=1,50
5003  FF(M,I) = 0.0
5004  GG(M,I) = 0.0
50    DO 77 N=1,NCTOFF
501   MYMAX = MYMIN+NMAX-N
51      OUTPUT ICUT,151,N
52      OUTPUT ICUT,152
53    DO 73 MY=MYMIN,MYMAX
54      OUTPUT ICUT,153,MY
540   MYZ = MY-MYMIN+1
541   ITTMAX = MYMAX-MY+1
55    DO 73 ITT=1,ITTMAX
56    IT = ITT-1
561   OUTPUT ICUT,154,IT
562   NMY = MY+IT
563   NMYZ = NMY-MYMIN+1
57    DO 73 K=1,2
58    GO TO (59,64),K
59      OUTPUT ICUT,155
61    FKEEP = YRGAIN(MY,IT,1)+A*FF(MYZ,ITT)
62    FREPL = R-P+YRGAIN(NMY,0,1)+A*FF(NMYZ,1)
63    IF (FKEEP-FREPL) 633,633,631
631   F(MYZ,IT) = FKEEP
6311  PROFIT = FKEEP
632   GO TO 70
633   F(MYZ,IT) = FREPL
6331  PROFIT = FREPL
634   GO TO 72
64      OUTPUT ICUT,156
66    GKEEP = PI*YRGAIN(MY,IT,1)+(1.0-PI)*YRGAIN(MY,IT,2)
```

```
     X        +A*(PI*FF(MYZ,ITT)+(1.C-PI)*GG(MYZ,ITT))
67    GREPL = R-P+PI*YRGAIN(NMY,0,1)+(1.0-PI)*YRGAIN(NMY,0,2)
     X        +A*(PI*FF(NMYZ,1)+(1.0-PI)*GG(NMYZ,1))
68    IF (GKEEP-GREPL) 683,683,681
681   G(MYZ,IT) = GKEEP
6811  PRCFIT = GKEEP
682   GC TO 70
683   G(MYZ,IT) = GREPL
6831  PRCFIT = GREPL
684   GC TO 72
7C      CLTPLT ICUT,157
71    GC TO 73
72      CUTPLT ICUT,158
73      OUTPUT ICUT,159,PRCFIT
74    DC 77 I=1,50
75    DC 77 M=1,5C
76    FF(M,I) = F(M,I)
77    GG(M,I) = G(M,I)
78    IF DIVICE CHECK 781,783
781   OUTPUT ICUT,782
782   FCRMAT (26HCDIVICE CHECK INDICATCR ON)
783   CALL EXIT
      END(1,1,C,C,0,0,1,0,C,0,0,0,0,0,0)
```

```
      FUNCTION YRGAIN(MYS,ITS,KS)
      COMMON ACAP,B,C,D,FR,MYMIN
C
1     T = ITS
2     YDEL =MYS-MYMIN
3     YRGAIN = (ACAP+B*(YDEL))*EXPF(-T/(C+D*YDEL))
4     GO TO (5,6),KS
5     YRGAIN = YRGAIN*FR
6     RETURN
      END
```

```
•      XEQ

ENTRY POINTS TO SUBROUTINES REQUESTED FROM LIBRARY,
    (ACTMR      (FPT)       (TSHM)      (RTN)       (STHM)      (FIL)       EXIT        EXP

LOADING MAP, ENTRY POINT ADDRESSES.      (00000 = MAIN PROGRAM.  - ADDRESS INDICATES UNUSED ENTRY POINT.)

    000000 00153    YRGAIN 24755    (FPT)  25056    EXP    25143    (KICK) 25231    (SPH) -25312    (STHD)-25316

    (STHM) 25313    (STH) -25306    (CSH) -25444    (TSHM) 25445    (TSH) -25440    (RDC)  25525    (RER)  25503

    (WTC)  25604    (WER)  25553    (RTN)  27470    (FIL)  27457    (IOH)  25635    (TCO)  30000    (TEF)  27777

    (RCH)  27776    (ETT)  27775    (REW)  27771    (WEF)  27770    (BSR)  27767    (WRS)  27766    (RDS)  27765

    (IOS)  27626    (RUN) -27774    (SDL) -27773    (SDH) -27772    (TRC)  30001    (IOU)  30021    (WOT)  30074

    BOBK  -30662    (EXEM) 30107    CLKOUT-30754    ERRDMP-30754    ENDJOB-30754    RETURN-30754    EXIT   30754

    (TALL)-31046    (ACTMR-31001    (TES)  31051

LOWEST UNUSED CELL      3.052

LOWEST COMMON CELL      77453

              NO SYSTEM TAPE ERRORS ENCOUNTERED DURING THIS JOB.

       EXECUTION                                        TIME 111030

NMAX = 28
NCTOFF = 10
NYMIN = 1945
PI =  0.50000000E 00,R =  0.50000000E 06,P =  0.40000000E 07,A =  0.90000000E 00
ACAP =  0.50000000E 06,B =  0.50000000E 05,C =  0.50000000E 01,D =  0.50000000E 00,FR =  0.70000000E 00
```

FOR PROCESSES OF DURATION 1 YEARS

------------ STATES ------------			OPTIMAL DECISION	OPTIMAL EXPECTED RETURN
AVE.YEAR PRESENT EQUIP'T.PURCHASED	PRESENT AGE OF EQUIP'T.	LEVEL OF COMPETITION		
1945	0	HEAVY	KEEP	3.50E 05
		NORMAL	KEEP	4.25E 05
	1	HEAVY	KEEP	2.87E 05
		NORMAL	KEEP	3.48E 05
	2	HEAVY	KEEP	2.35E 05
		NORMAL	KEEP	2.85E 05
	3	HEAVY	KEEP	1.92E 05
		NORMAL	KEEP	2.33E 05
	4	HEAVY	KEEP	1.57E 05
		NORMAL	KEEP	1.91E 05
	5	HEAVY	KEEP	1.29E 05
		NORMAL	KEEP	1.56E 05
	6	HEAVY	KEEP	1.05E 05
		NORMAL	KEEP	1.28E 05
	7	HEAVY	KEEP	8.63E 04
		NORMAL	KEEP	1.05E 05
	8	HEAVY	KEEP	7.07E 04
		NORMAL	KEEP	8.58E 04
	9	HEAVY	KEEP	5.79E 04
		NORMAL	KEEP	7.03E 04
	10	HEAVY	KEEP	4.74E 04
		NORMAL	KEEP	5.75E 04
	11	HEAVY	KEEP	3.88E 04
		NORMAL	KEEP	4.71E 04
	12	HEAVY	KEEP	3.18E 04
		NORMAL	KEEP	3.86E 04
	13	HEAVY	KEEP	2.60E 04
		NORMAL	KEEP	3.16E 04
	14	HEAVY	KEEP	2.13E 04
		NORMAL	KEEP	2.58E 04
	15	HEAVY	KEEP	1.74E 04
		NORMAL	KEEP	2.12E 04
	16	HEAVY	KEEP	1.43E 04
		NORMAL	KEEP	1.73E 04
	17	HEAVY	KEEP	1.17E 04
		NORMAL	KEEP	1.42E 04
	18	HEAVY	KEEP	9.56E 03
		NORMAL	KEEP	1.16E 04
	19	HEAVY	KEEP	7.83E 03
		NORMAL	KEEP	9.51E 03
	20	HEAVY	KEEP	6.41E 03
		NORMAL	KEEP	7.78E 03
	21	HEAVY	KEEP	5.25E 03
		NORMAL	KEEP	6.37E 03
	22	HEAVY	KEEP	4.30E 03
		NORMAL	KEEP	5.22E 03
	23	HEAVY	KEEP	3.52E 03
		NORMAL	KEEP	4.27E 03
	24	HEAVY	KEEP	2.88E 03
		NORMAL	KEEP	3.50E 03
	25	HEAVY	KEEP	2.36E 03
		NORMAL	KEEP	2.86E 03

	26	HEAVY	KEEP	1.93E 03
		NORMAL	KEEP	2.34E 03
	27	HEAVY	KEEP	1.58E 03
		NORMAL	KEEP	1.92E 03
1946	0	HEAVY	KEEP	3.85E 05
		NORMAL	KEEP	4.67E 05
	1	HEAVY	KEEP	3.21E 05
		NORMAL	KEEP	3.90E 05
	2	HEAVY	KEEP	2.68E 05
		NORMAL	KEEP	3.25E 05
	3	HEAVY	KEEP	2.23E 05
		NORMAL	KEEP	2.71E 05
	4	HEAVY	KEEP	1.86E 05
		NORMAL	KEEP	2.26E 05
	5	HEAVY	KEEP	1.55E 05
		NORMAL	KEEP	1.88E 05
	6	HEAVY	KEEP	1.29E 05
		NORMAL	KEEP	1.57E 05
	7	HEAVY	KEEP	1.08E 05
		NORMAL	KEEP	1.31E 05
	8	HEAVY	KEEP	8.99E 04
		NORMAL	KEEP	1.09E 05
	9	HEAVY	KEEP	7.50E 04
		NORMAL	KEEP	9.10E 04
	10	HEAVY	KEEP	6.25E 04
		NORMAL	KEEP	7.59E 04
	11	HEAVY	KEEP	5.21E 04
		NORMAL	KEEP	6.33E 04
	12	HEAVY	KEEP	4.34E 04
		NORMAL	KEEP	5.28E 04
	13	HEAVY	KEEP	3.62E 04
		NORMAL	KEEP	4.40E 04
	14	HEAVY	KEEP	3.02E 04
		NORMAL	KEEP	3.67E 04
	15	HEAVY	KEEP	2.52E 04
		NORMAL	KEEP	3.06E 04
	16	HEAVY	KEEP	2.10E 04
		NORMAL	KEEP	2.55E 04
	17	HEAVY	KEEP	1.75E 04
		NORMAL	KEEP	2.13E 04
	18	HEAVY	KEEP	1.46E 04
		NORMAL	KEEP	1.77E 04
	19	HEAVY	KEEP	1.22E 04
		NORMAL	KEEP	1.48E 04
	20	HEAVY	KEEP	1.01E 04
		NORMAL	KEEP	1.23E 04
	21	HEAVY	KEEP	8.46E 03
		NORMAL	KEEP	1.03E 04
	22	HEAVY	KEEP	7.05E 03
		NORMAL	KEEP	8.56E 03
	23	HEAVY	KEEP	5.88E 03
		NORMAL	KEEP	7.14E 03
	24	HEAVY	KEEP	4.90E 03
		NORMAL	KEEP	5.95E 03
	25	HEAVY	KEEP	4.09E 03
		NORMAL	KEEP	4.96E 03
	26	HEAVY	KEEP	3.41E 03
		NORMAL	KEEP	4.14E 03

1947	0	HEAVY	KEEP	4.20E 05
		NORMAL	KEEP	5.10E 05
	1	HEAVY	KEEP	3.56E 05
		NORMAL	KEEP	4.32E 05
	2	HEAVY	KEEP	3.01E 05
		NORMAL	KEEP	3.65E 05
	3	HEAVY	KEEP	2.55E 05
		NORMAL	KEEP	3.09E 05
	4	HEAVY	KEEP	2.16E 05
		NORMAL	KEEP	2.62E 05
	5	HEAVY	KEEP	1.83E 05
		NORMAL	KEEP	2.22E 05
	6	HEAVY	KEEP	1.55E 05
		NORMAL	KEEP	1.88E 05
	7	HEAVY	KEEP	1.31E 05
		NORMAL	KEEP	1.59E 05
	8	HEAVY	KEEP	1.11E 05
		NORMAL	KEEP	1.34E 05
	9	HEAVY	KEEP	9.37E 04
		NORMAL	KEEP	1.14E 05
	10	HEAVY	KEEP	7.93E 04
		NORMAL	KEEP	9.63E 04
	11	HEAVY	KEEP	6.71E 04
		NORMAL	KEEP	8.15E 04
	12	HEAVY	KEEP	5.68E 04
		NORMAL	KEEP	6.90E 04
	13	HEAVY	KEEP	4.81E 04
		NORMAL	KEEP	5.84E 04
	14	HEAVY	KEEP	4.07E 04
		NORMAL	KEEP	4.95E 04
	15	HEAVY	KEEP	3.45E 04
		NORMAL	KEEP	4.19E 04
	16	HEAVY	KEEP	2.92E 04
		NORMAL	KEEP	3.54E 04
	17	HEAVY	KEEP	2.47E 04
		NORMAL	KEEP	3.00E 04
	18	HEAVY	KEEP	2.09E 04
		NORMAL	KEEP	2.54E 04
	19	HEAVY	KEEP	1.77E 04
		NORMAL	KEEP	2.15E 04
	20	HEAVY	KEEP	1.50E 04
		NORMAL	KEEP	1.82E 04
	21	HEAVY	KEEP	1.27E 04
		NORMAL	KEEP	1.54E 04
	22	HEAVY	KEEP	1.07E 04
		NORMAL	KEEP	1.30E 04
	23	HEAVY	KEEP	9.09E 03
		NORMAL	KEEP	1.10E 04
	24	HEAVY	KEEP	7.69E 03
		NORMAL	KEEP	9.34E 03
	25	HEAVY	KEEP	6.51E 03
		NORMAL	KEEP	7.91E 03
1948	0	HEAVY	KEEP	4.55E 05
		NORMAL	KEEP	5.52E 05
	1	HEAVY	KEEP	3.90E 05
		NORMAL	KEEP	4.74E 05
	2	HEAVY	KEEP	3.34E 05
		NORMAL	KEEP	4.06E 05
	3	HEAVY	KEEP	2.87E 05

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